VIBRATIONS FORCED BY F\*SIN(\*t) OR F\*COS(\*t) v1.11

The program finds for the differential equation ( d. e.) of a forced vibration given as

m\*d2y/dt2 + d\*dy/dt + c\*y = F0\*cos(\*t) or m\*d2y/dt2 + d\*dy/dt + c\*y = F0\*sin(\*t)

or a combination of both ….. = F0\*cos(\*t) + F1\*sin(\*t)

the exact general solution and replaces the constants of integration C1, C2 by numerical values dependant to preset initial conditions t0, y0(t0), y'0(t0).

The factors are in particular:

m: mass of system [kg], d: damping factor [kg/s] and c: constant of spring [N/m], F0, F1: external forces [N) and : frequency of the excitation [1/s].

This equation is customarily written as

d2y/dt2 + 2\*\*dy/dt + 2\*y = K0\*cos(\*t) or d2y/dt2 + 2\*\*dy/dt + 2\*y = K0\*sin(\*t),

where: 2\* d/m,2 c/m, K0  = F0/m

*In the graph-screen the x-axis represents the t-abscissa of the calculation*.

The whole program consists of four sub-programs, which all have to be downloaded to the calculator: FORCDVIB offers the selection for one of the following three sub-programs, FOVIBCOS handles the right-hand side given as F0\*cos(\*t), FOVIBSIN does the same for F0\*sin(\*t) as applied force to the system and FVCOSSIN handles the combination case.

Start FORCDVIB and select one of the three offered possibilities. Then enter the quantities of m, d, c, F0 ( additionally F1 for case three ) and . Following, the program asks for the initial conditions t0, y0(t0), y'0(t0). Finally the program prompts for Xmax as time interval to be displayed in the graph-screen. After the input of these values the calculation is started and the program displays the homogeneous part yh of the equation, its curve in LIGHTBLUE and an information about the state of the system (overdamped, aperiodic borderline state, underdamped ) in the graph screen. For an underdamped system you will find on the top of the screen the solution for yh as function of *sin*  and *cos*, while at the bottom the equivalent function only in *sin* for FOVIBSIN or *cos* (FOVIBCOS ) is depicted. Press [enter] to see the particular part yp of the solution (MAGENTA) and the superimposed curve of both solutions (DARKBLUE). Press [enter] again and the program asks to enter a new time interval 0<=t<=Xmax (1) or to quit program execution (2). After having quitted the calculation, you may press the [graph]-key, then [trace] and scroll up/down to the curves and/or left/right to any time-values.

*EXAMPLE:*

A body with mass m = 20 kg and a spring factor c = 1620 N/m of the system is oscillating in a damping fluid with d = 35 kg/s, excited by a vibrating force F0 = 15 N \* cos (13\*t). Find the equation, the course of the oscillation and y at t=4 . The movement starts at x0(=t0)=0 motionless from the origin (y(0)=y’(0)=0).

Start FORCDVIB, select line 1 ( cos - term) and key in the appropriate values 20, 35, 1620, 15, 13 to each prompt. Then continue the input by 0, 0, 0 for the initial conditions. To scrutinize a time interval of 6 seconds, key in this value to “Xmax: ” ( Xmin is set to -0.1 !) and press [enter].

The first contribution of the solution is the homogeneous part ( values rounded here !):

yh =e-0.875\*t \* (-0.002217\*sin(8.95736\*t) + 0.007989\*cos(8.95736\*t)

on the top oft the screen and at the bottom:

yh =0.0082907\*e-0.875\*t \*cos(8.95736\*t+0.2707)

Furthermore the display informs about the state of the system ( system underdamped ).

Now press [enter] to get the particular solution dependent on the right-hand side of the d. e.:

yp = 0.00825\*cos(13\*t-2.8886), A(w=0)=F0/c=0.009259

The amplitude A is a function dependent on the excitation frequency :

A= F0 / (m\*sqrt((2 - 2 ) 2 +4 \* 2 2 ))

For =0 the quantity of A simplifies to F0/c representing the stationary solution . This value is displayed after yp.

The graph of the total solution y = yh + yp is drawn in DARKBLUE.

Press [enter] again and enter a new time interval (1), e.g. 5 for the first five seconds of the oscillation or quit program execution by selecting (2).

Press [graph], then [trace] and select the DARKBLUE curve by the up/down keys. Then key in *4*

[enter] and at the bottom of the screen you will find y = 0.0033327 [m].

Now restart the program and change d to 1.5 and to 9. As =sqrt (c/m) = 9, both frequencies have the same size. By the selection of a weak damping factor the vibrations will build up and you have created a resonance disaster !

*REFERENCES:*

“Mathematik für Ingenieure und Naturwissenschaftler“, L. Papula; F. Vieweg & Son Publishers, 1997

“Technische Mechanik”, J. Dankert/H. Dankert; Vieweg + Teubner publisher’s 20009

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