HOMOGENEOUS DIFFERENTIAL EQUATION of 2nd ORDER v1.03

The program finds for a homogeneous differential equation ( d. e.) of 2nd order given as

(*I*) y''+A\*y'+B\*y=0 the exact general solution and replaces the constants of integration C1, C2 by numerical values dependent to preset initial conditions x0, y0(x0), y'0(x0).To scrutinize the

d. e. of a damped unforced vibration (A>0 and B>0) given as:

m\*d2y/dt2 + b\*dy/dt + c\*y = 0, (*II*)

where: m: mass of system [kg], b: damping factor [kg/s] and c: constant of spring [N/m], the input of A in the previous equation (*I*) has to be done as b/m and for B as c/m (cf. example), so that (*II*)

looks like (*I*): d2y/dt2 + (b/m)\*dy/dt + (c/m)\*y = 0 (*III*)

Internally the program replaces the variable of time t in this case by the variable x in the results.

Start HOMDEQU2, then, on the prompt for A and B, enter the appropriate values. Now the program asks for the initial conditions x0, y0(x0), y'0(x0). Two possibilities to continue are provided: to get the general solution containing the constants C1 and C2, enter -99 on the prompt for x0. The program displays the general solution in the graph-screen. To find the particular solution, press keys 2nd quit, then enter. The program starts again. Now key in the same values of A and B, then the quantities for x0, y0, y’0. The particular solution is now displayed including the graph of y.

Besides, if A>0 and B>0, the program depicts the state of the oscillation in form of the variable

D=A/(2\*sqrt(B)) as overdamped ( D>1), aperiodic borderline state (D=1) or underdamped (D<1). In this case, additionally the time period Td of the oscillation and the logarithmic decrement =

ln (y(t)/y(t+Td)) are indicated.

*EXAMPLE:*

A spring with mass m = 10 kg and constant c=250 N/m is oscillating in a fluid with damping factor b= 80 kg/s under initial conditions y(0)= 0.6 m, y’(0)=0 m/s. Find the general and the particular solution y(t) of the system.

Start HOMDEQU2, then key in 8 (=80/10) for A and 25 (=250/10) for B. Now enter -99 for x0.

The general solution is y= e-4\*x \*( C1\*sin(3\*x)+C2\*cos(3\*x)). Now press keys 2nd quit, then enter to restart the program. Key in again 8 and 25 for A and B, then on the prompts: 0 for x0,

.6 and 0 for y0 and y’0.

The graph-screen depicts, besides the graph of the function, on the top of the display the particular solution as:

y= e-4\*x \*( 0.8\*sin(3\*x)+0.6\*cos(3\*x)). At the bottom of the screen the equivalent result

as a function of sin only including phase-shift is presented:

= 1\*e-4\*x \*sin(3\*x+0.6435011) and on the right side: Td=2.09439, =-8.377, D=0.8.

*REFERENCES:*

“Mathematik für Ingenieure und Naturwissenschaftler“, L. Papula; F. Vieweg & Son Publishers, 1997

„Differentialgleichungen“ , F. Ayres jr. ; Schaum’s Outline, McGraw-Hill Book Company Ltd. 1991

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