Patterns

I'm a tile layer and a very amateur lover of math—especially geometry. Tiling a shower is an endeavor with geometric patterns—of laying one tile next to another in a regular design. Sometimes a designer will choose tiles that weren't meant to go together dimensionally; yet the tile installer's job is to make it work, because the designer likes the colors. So the installer sets to work trying to find some commonality between the different tiles' dimensions and design—to bring the whole project together to a satisfying conclusion.

Prime numbers are like that. They seem to be totally unrelated with no commonality whatsoever, beyond the fact that they can't be divided evenly by any other number except 1 and themselves. Prime numbers seem to be the symbolic equivalent to a desert full of rocks after an earthquake shakes them into place randomly until they find a natural fit. At least it appears that random.

Yet there is that one nasty thing that counters that notion: Numbers march one after another: continually, eternally in the same order—always; and never wavering.

We assign meaning to a number by putting it on a number line.

… -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 ….

But why is a particular number that appears where it does either prime or composite?

A tile layer is tasked with putting a design together so that the result looks natural and pleasing—everything good. He looks to find *some little* place to start. But where did God start with the numbers' constellation? With primes, people have pondered this for millennia. So who am I? … just a tile layer who looks in awe and wonder at the constellation of numbers and tries to figure out how God put them together so that I might bear his image or reflect his glory—even if just a dim reflection.

One of the rules of good practice in tiling is to find a balance between symmetry and randomness. Numbers do this by putting an odd integer next to an even. That is at a regular cadence—a boring symmetry. Primes don't seem to march to any regular cadence, but they do seem to have some symmetry, but we can't quite put our fingers on it—and it's not boring.

But there's something there. When we look at the primes, this is what we have: 2, 3, 5, 7, 11, 13, 17, 19, 23 …. What *do* we see in this? Well it's not satisfying to the overly-anal personality that befits most mathematicians. Yet *after* 3 we see that the numbers fall in the addition-pattern of 2 + 4 + 2 + 4 + 2 + 4 …. And then we run into a demon that must be cast out: 25! It messes up our neat little ditty! And we don't have to go far until we find more demons: 35 and 49. And they keep coming, not diminishing, but increasing: 55, 65, 77, 85, 91, 95 …. Mathematicians throughout the corridors of time have screamed a collective, "AHHH!" And worse!

We want something nice and tidy, but find what seems to be chaos. Yet the tile installer in me wants to find some approach to lay these numbers in a way that displays their beauty. Sometimes in laying out tiles you just have to try something and make it work. But still you want to find some small commonality to start with. There's one trick in tiling: You can literally "turn a corner." Tiling around a corner is an opportunity to make an adjustment—even something as drastic as reversing the pattern!

So I began looking for some small commonality with 2 + 4 + 2 + 4 …. I wanted something better. I noticed that the first two primes (2 and 3) were out of sync—but they themselves had something to do with that fact. *Two* eliminates even numbers from the possibility of primality. *Three* eliminates all the multiples of 3, half of which are even. That's what sets up the 2 + 4 + 2 + 4 …. These are where the *prime candidates* are located; but obviously not all are prime.

This leaves the fact that all primes (other than 2 or 3) have a remainder of 1 or 5 when divided by 6.

But then I "turned a corner;" and in so doing I considered a mathematical *anathema*: What if we "momentarily" allow negative prime candidates? Well, they may not really be negative, but just counted backwards, or in the mirror—or around the corner. I suppose numbers don't really care which way they march.

If primes aren't negative, some at least behave like negatives—in a Ying, Yang sort of way. Take a look at this set of numbers:

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{ | … -35, -29, -23, -17, -11, -5, 1, 7, 13, 19, 25, 31, 37 … | }

This is an infinite set of numbers, whose absolute value contains the set of all primes—except those two demonic ones (2 and 3). Yet not all are prime. These prime candidates are spaced 6 apart. And they are all congruent to 1 MOD 6. Let's call them P.

What's nice about this is the sense of regularity joined to randomness. These numbers march like the number line. And, indeed, they align with the number line:

P: …	-35	-29	-23	-17	-11	-5	1	7	13	19	25	31	37	43	49 …
Q: …	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8 …

Let's call the numbers on the number line Q, since they queue up the prime candidates P:

6Q + 1 = P and $\frac{P-1}{6} = Q$.

But wait! There's more! Here's how the P's left of zero act like negative numbers: When you multiply a negative P by a negative P, you get a positive P ($-5 \times -5 = 25$). When you multiply a negative P times a positive P, you get a negative P ($-5 \times 7 = -35$). When you multiply a positive P times a positive P, you get a positive P ($7 \times 7 = 49$).

And more: Since we have a number line, let's extend it to the Cartesian Plane:

				Y axis				
	-71	-46	-21	4	29	54	79	
	-425	-275	-125	25	175	325	475	
	-54	-35	-16	3	22	41	60	
	-323	-209	-95	19	133	247	361	
	-37	-24	-11	2	15	28	41	
	-221	-143	-65	13	91	169	247	
	-20	-13	-6	1	8	15	22	
	-119	-77	-35	7	49	91	133	
X axis	-3	-2	-1	0	1	2	3	Q
	-17	-11	-5	1	7	13	19	P
	14	9	4	-1				
	85	55	25	-5		This quadrant		
	31	20	9	-2		is the same as		
	187	121	55	-11		diagonal quad		
	48	31	14	-3				
	289	187	85	-17				
	65	42	19	-4				
	391	253	115	-23				

When we fill in the grid with the multiples to infinity, we get the set of composite numbers that are not divisible by 2 or 3. And these composites are all in the set of P.

Since every composite is also of the form 6Q + 1 = P, every composite in the grid also falls some place on the X or Y axes. So Q = some x or y on the Cartesian axes.

Moreover, we can take (6x + 1)(6y + 1) = P and expand it to 36xy + 6x + 6y +1. Substituting 6Q + 1 for P, we can reduce the equation to Q = 6xy + x + y. When x and y are both integers, they give us P's coordinates in the Cartesian plane. Solving this equation for its integer solutions gives us the Q position of P's divisors (which will be all of its prime divisors and all of its composite divisors. Taking these integer solutions and multiplying each by 6x + 1 or 6y + 1 gives us the divisors.

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For example, if P = 91, then Q = 15, since $\frac{91-1}{6} = 15$.

Then, 91 = 36xy +6x + 6y + 1, reduces to
15 = 6xy +x + y.

Solving for x: $x = \frac{15-y}{6y+1}$.

y	x
0	15
1	2
2	1
15	0

In either case, the integer solutions are x = 0, 1, 2, 15; and y = 0, 1, 2, 15.

These solutions map to the divisors of 91:

0 maps to 1 (6 * 0 + 1 = 1)

1 maps to 7 (6 * 1 + 1 = 7)

2 maps to 13 (6 * 2 + 1 = 13)

15 maps to 91 (6 * 15 + 1 = 91)

The divisors of 91 are 1, 7, 13, and 91.

TI-84 BASIC code	Explanation
1→I	Initialize counter
1→dim(L1)	Dimension List variable
{0}→L1	Initialize List variable
Input “INTEGER?”, N	User input of number to factor
If N<1 or fPart(N)≠0: Then	Must be positive integer
Disp “ENTER A”	
Disp “POSITIVE”	
Disp “INTEGER”	
Stop	
End	
N→M	Store the number for later
N→L	
While remainder(N,2)=0	Cast out 2’s from the number
N/2→N	
2→L1(I)	Store 2 in the list every time it divides the number
I+1→I	Increase the counter
End	

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While remainder(N,3)=0	Cast out 3's from the number
N/3→N	
3→L1(I)	Store 3 in the list and increase the counter
I+1→I	(After 2's & 3's are cast out, the remaining number will fit into the formula)
End	
round((N^.5)/6),0)→S	1/6 the square root sets the upper search parameter
If remainder(N,6)=5: -N→N	Change number to negative if $N \equiv 5$ MOD 6
(N-1)/6→Q	Q is the number's position on the number line
For(X,-S,S)	Loop for integer solutions from negative 1/6 square root to positive 1/6 square root
(Q-X)/(6X+1) →Y	
If fPart(Y)=0: Then	If Y has an integer solution, then X does too
abs(6X+1) →L1(I)	Convert integer solutions to divisors and store in List 1
I+1→I	
abs(6Y+1) →L1(I)	
I+1→I	
End	
End	
SortA(L1)	Ascending sort on list
2→I	Initialize counter
{1}→L2	Initialize new list
Dim(L1) →K	Store the number of elements in first list
1→H	Initialize counter
For (J,1,2)	Skip 1's in list
If L1(J)= 1: H+1→H	
End	
For(J,H,K)	
While fPart(M/L1(J))=0	Loop through divisors to find prime factors
L1(J) →L2(I)	Store prime factors in List 2
M/L1(J) →M	
I+1→I	
End	
End	
If dim(L2)>2:L→L2(I)	Add the original number to the end of List 2 if not prime
If dim(L2)=2: Disp "PRIME"	If the list only has two elements, the number is prime
For (J,1,dim(L2))	Display List 2, the prime factors, and the original number
Disp L2(J)	
End	

Note: L1 has all the divisors; L2 has only prime divisors, plus 1 and the original number that was entered.

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Stop

This code could be optimized. One idea would be to restart its search parameters after it finds a divisor, by first dividing out that divisor. This would diminish the search range, which would be especially effective on large numbers. Of course, if the divisors are only the square root or a number close to the square root, then this would not increase the speed. See my other factoring programs (MODFAC and MODFACTR) for this feature.

Also, I'm sure there are better algorithms that could search more efficiently. I would love to have feedback from any number theorist who can see beyond my limited understanding. You may email me at curtis.liechty@gmail.com .

One final note: The graph of this equation is a hyperbola on a right cone that is rotated 45°. When P is positive, the cone is mostly in the SW and NE quadrants, with different possible integer solutions in both quadrants. When P is negative, the cone is in the NW and SE quadrants, with the same integer solutions in both quadrants—but with the x and y flipped. The center of the cone on a 3D grid is at $(-\frac{1}{6}, -\frac{1}{6}, 0)$. The asymptotes go through the center parallel to the x and y axes.

When P is positive: The hyperbola's semi-major axis length (a) and the semi-minor axis length (b) are both $\frac{\sqrt{2P}}{6}$ units. The z value of the entire hyperbola on the 3D graph is $\frac{\sqrt{2P}}{12}$.

One vertex is located on the x,y axis at $(\frac{\sqrt{P}}{6} - \frac{1}{6}, \frac{\sqrt{P}}{6} - \frac{1}{6})$.

The foci are located at $\frac{\sqrt{P}}{3}$ (c) units (diagonally) from the center.

The x and y intercepts are at $\frac{P-1}{6}$.