Wrkenrgy()   Applications of the Work-Energy Principle   3-14-2011

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This program uses the Work-Energy principle to solve problems involving any number of forces (in a plane) applied to an object or a system of objects, pulleys, and springs. It treats objects as point particles, that is, it will not treat rotation (except pulleys). If a pulley is not assumed to be massless, it is treated as a separate object in the system, and the effects of its mass, radius and inertia are included.

The program will handle some cases of motion on a curved path if friction is the only non-constant force which does work on the object. Either numerical or symbolic solutions can be found, although the symbolic solutions are sometimes slow. Equations are displayed as the program runs and are also copied to the home screen.

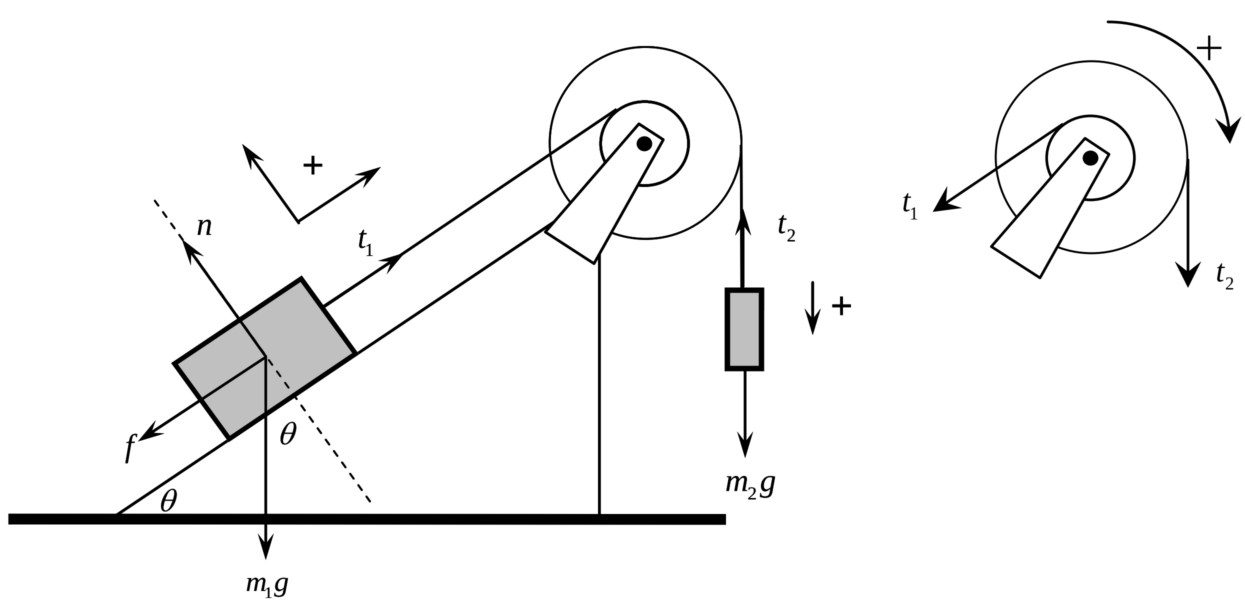
Any coordinate system can be used, although it is probably easier, for each object, to choose one axis parallel to the direction of motion.

When the number of unknowns exceeds the number of equations, you can choose which to solve for: the rest are treated as parameters. In general, the program cannot be used to solve for angles.

All solutions are copied to the home screen (using Copyto_h(), by Samuel Stearley) so they can be more easily used in further calculations.

Partsand(), Partsxll(), Varlist(), and Copyto_h() are used in the main program. Place all programs in the same folder, then run Wrkenrgy().

Example 1: A block of mass $m_1 = 20\ \text{kg}$ is pulled upward along a 20° incline by a hanging mass $m_2 = 15\ \text{kg}$ and the rope and pulley system shown. The coefficient of friction on the incline is $\mu = 0.2$. The pulley has mass $m_p = 3\ \text{kg}$, moment of inertia $I_p = 0.02\ \text{kg}\cdot\text{m}^2$, inner radius $r_1 = 0.05\ \text{m}$, and outer radius $r_2 = 0.10\ \text{m}$. If the system starts from rest, find the speed of each block and the tension in each rope, after m_1 has moved 0.5 m along the incline.



Choose: MKS, Normal gravity, 3 Objects, 0 Springs

Object 1: No Not a pulley

Yes moves in a straight line

1 force other than gravity, friction, springs, and normal

$m1 = 20$

$\theta g = -110$

$vo = 0$

$v = v1$

$\Delta x = .5$

$\Delta y = 0$

$\theta = 0$

$n = n1$

$\theta n = 90$

$\mu = .2$

$\theta f = 180$

$f1 = t1$ (or leave as $f1$)

$\theta 1 = 0$

Equations displayed:

$10 \cdot v1^2 = -.1 \cdot (n1 - 5 \cdot (t1 - 67.0359))$

$0 = .5 \cdot (n1 - 184.18)$

Continue

Object 2: No Not a pulley

Yes moves in a straight line

1 force other than gravity, friction, springs, and normal

Enter values:

$m = 15$

$\theta g = 0$

$vo = 0$

$v = 2 \cdot v1$ (travels twice as far since $r2 = 2 \cdot r1$)

$\Delta x = 1$

$\Delta y = 0$

$\theta = 0$

$n = 0$ (since no contact with a surface)

$\theta n = 90$ (value doesn't matter since $n = 0$)

$\mu = 0$

$\theta f = 180$ (value doesn't matter since $\mu = 0$)

$f1 = t2$

$\theta 1 = 180$

Equations displayed:

$30 \cdot v1^2 = 147 - t2$

Continue

Object 3: Yes A pulley

$I = .02$

$\omega o = 0$

$\omega = v1 / .05$ ($\omega = v1 / r1 = v2 / r2$)

$\Delta\theta = .5 / .05$ ($\Delta\theta = x / r1$)

$T2 = t2$

$L2 = .1$

$T1 = t1$

$L1 = .05$

Equations displayed:

$-.5 \cdot (t1 - 2 \cdot t2) = 4 \cdot v1^2$

Continue

Solutions:

$n1 = 184.18$

$t1 = 147.083$

$t2 = 82.1836$

$v1 = 1.46987$

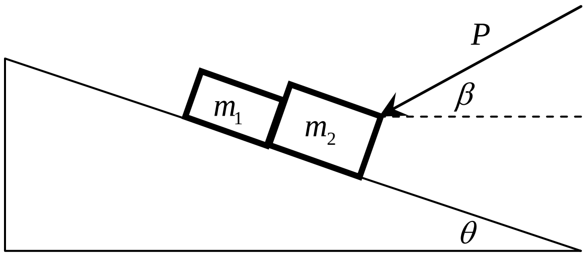
Quit

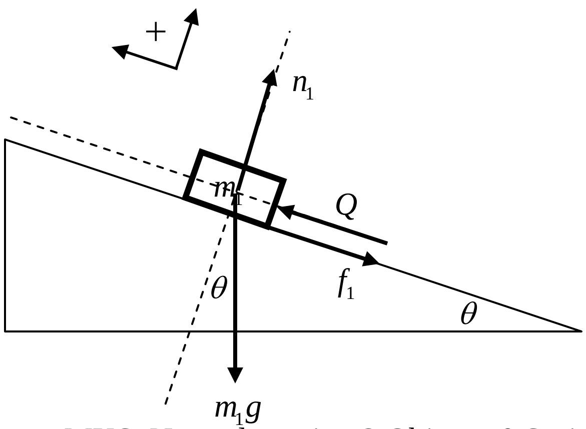
Then:

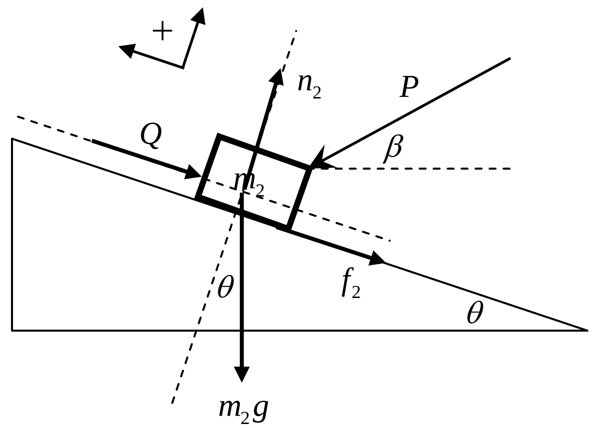
$v2 = 2v1 = 2.93974$

Solutions are calculated and copied to the home screen, along with the equations.

Example 2: Two blocks, $m_1 = 20$ kg and $m_2 = 30$ kg, are pushed up a $15°$ incline by a force $P = 400$ N applied at an angle $\beta = 20°$ as shown. The coefficient of friction between the blocks and the incline is $\mu = 0.2$. If the blocks are moving initially at speed $v_o = 0.5$ m/s^2, determine the speed of the blocks after moving 2 m along the incline and the contact force each block exerts on the other.







Choose: MKS, Normal gravity, 2 Objects, 0 Springs
Object 1: No Not a pulley
Yes moves in a straight line
1 force other than gravity, friction, springs, and normal
Enter values:

$m = 20$

$\theta g = -105$

$vo = .5$

$v = v$

$\Delta x = 2$

$\Delta y = 0$

$\theta = 0$

$n = n1$

$\theta n = 90$

$\mu = .2$

$\theta f = 180$

$f1 = q$

$\theta 1 = 0$

Equations displayed:

$10 \cdot (v^2 - .25) = 2 \cdot (q - .2 \cdot (n1 + 253.643))$

$0 = 2 \cdot (n1 - 189.321)$

Continue

Object 2: No Not a pulley
Yes moves in a straight line
2 forces other than gravity, friction, and normal.

Enter values:

$\mu = .2$

$\theta f = 180$

$f1 = q$

$\theta 1 = 180$

$f2 = 400$

$\theta 2 = -35$

Equations displayed:

$15 \cdot (v^2 - .25) = -2 \cdot (q + .2 \cdot (n2 - 1257.84))$

$0 = 2 \cdot (n2 - 513.413)$

Continue

Solutions:

$v = 2.25244$

$q = 112.71$

$n1 = 189.321$

$n2 = 513.413$

Quit

Quit

The two previous examples could have been solved by using Newt2sys(), then using the results in acceler8() since the forces are constant, resulting in uniformly accelerated motion. The next three examples are cases in which there are variable forces, so that approach can't be used.

Example 3. A 10 kg block rests on the horizontal surface. The spring, which is not attached to the block, has a force constant of $k = 500$ N/m and is imitially compressed 0.2 m from C to A . After the block is released from rest at A, determine its speed when it passes point D. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.

Choose: MKS, Normal gravity, 1 Objects, 1 Spring
Spring #1

$k1 = 500$

$x1o = .2$

$x1 = 0$

Object 1: No Not a pulley
Yes moves in a straight line
0 force other than gravity, friction, springs, and normal
Enter values:

$m = 10$

$\theta g = -90$

$vo = 0$

$v = v$

$\Delta x = .4$

$\Delta y = 0$

$\theta = 0$

$n = n$

$\theta n = 90$

$\mu = .2$

$\theta f = 180$

Equations displayed:

$5 \cdot v^2 = 10 - .08 \cdot n$

$0 = .4 \cdot (n - 98)$

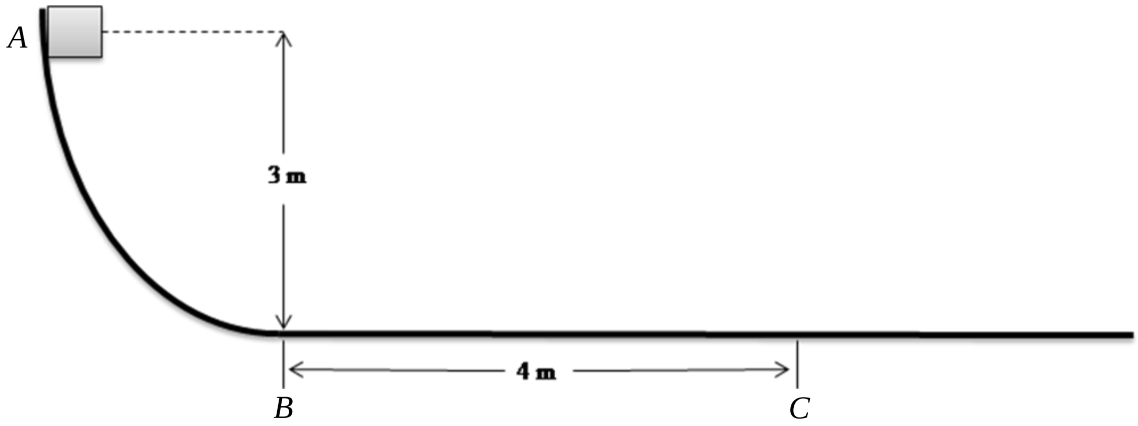
Continue

Solutions:

$v = .657267$

$n = 98$

Example #4. A 10 kg particle slides down a curved surface onto a horizontal surface. The coefficient of kinetic friction on the horizontal surface is $\mu_k = 0.15$. The particle is released at a point 3 m above the horizontal surface. The particle comes to rest after sliding 4 m. Calculate the work done by friction, W_f, as the particle slides down the curved surface.



First, consider the motion on the horizontal surface to determine the speed of the particle at *B*.
Choose: MKS, Normal gravity, 1 Objects, 0 Springs
Object 1: No Not a pulley
Yes moves in a straight line
0 force other than gravity, friction, springs, and normal
Enter values:

$m = 10$	$n = n$	
$\theta g = -90$	$\theta n = 90$	
$vo = vo$	$\mu = .15$	Solutions:
$v = 0$	$\theta f = 180$	$n = 98$
$\Delta x = 4$	Equations displayed:	$vo = 3.42929$
$\Delta y = 0$	$-5 \cdot vo^2 = -.6 \cdot n$	Quit
$\theta = 0$	$0 = 4 \cdot (n - 98)$	
	Continue	

Now use this speed as the final speed on the curved surface.
Choose: MKS, Normal gravity, 1 Objects, 0 Springs
Object 1: No Not a pulley
No does not move in a straight line
No No forces other than gravity and friction do work on the object.

$m = 10$

$\theta g = -90$

$vo = 0$

$v = 3.42928$

Enter values: $\Delta y = -3$

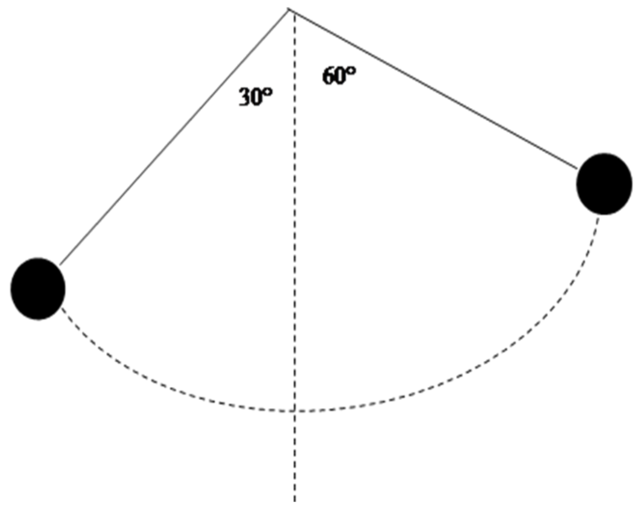
$W_f = wf$

Equations displayed:

$58.8001 = wf + 294$

$wf = -235.2$

Example #5. A simple pendulum consists of a small ball attached to a 2 m long string. The ball is pulled aside to an angle of 60° from vertical. It is given an initial speed of 1.5 m/s. Find its speed when it reaches the 30° position shown.
Choose: MKS, Normal gravity, 1 Objects, 0 Springs



Object 1: No Not a pulley
No does not move in a straight line
No No forces other than gravity and friction do work on the object.
Enter values:

$m = m$

$\theta g = -90$

$vo = 1.5$

$v = v$

$\Delta y = -2 \cdot (\cos 30° - \cos 60°)$

$W_f = 0$

Equations displayed:

$.5 \cdot m \cdot (v^2 - 2.25) = 7.1741 \cdot m$

TOO MANY UNKNOWNS

1Unknown Allowed

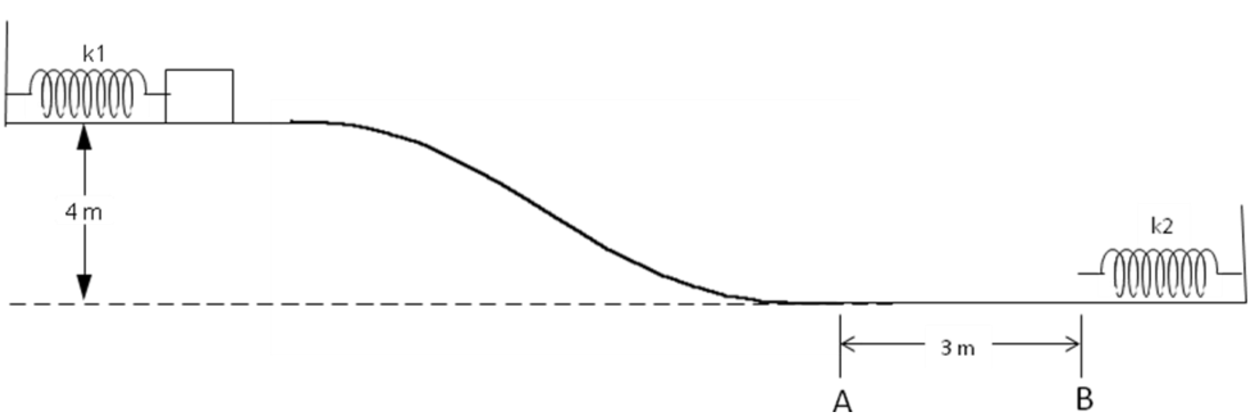
Delete any1

m, v Delete m

$v = 4.07409$

Quit

Example #6.
In the situation shown, the surface to the left of point A is frictionless. To the right of A, $\mu = 0.4$. The force constants of the springs are $k_1 = 400$ N/m and $k_2 = 800$ N/m. An 8 kg block is pushed against spring #1 so that the spring is compressed 0.300 m from its normal length. It is then released and it slides along the surface until it strikes spring #2 at B and compresses it. Determine how far the spring is compressed.



Since the object moves along a frictionless curved surface and then along a rough straight surface, we'll use the program twice, first along the curved surface to determine the speed at A, then again using that speed as the initial speed for the straight surface.

Choose: MKS, Normal gravity, 1 Objects, 1 Spring
Spring #1
$k1 = 400$

$x1o = .3$

$x1 = 0$
Object 1: No Not a pulley
No moves in a straight line
No forces other than gravity and friction do work
Enter values:
$m = 8$

$\theta g = -90$

$vo = 0$

$v = v$

$\Delta y = -4$

$Wf = 0$

Equations displayed:

$4 \cdot v^2 = 313.6$

Continue

$v = 8.85438$

Then run again

Choose: MKS, Normal gravity, 1 Objects, 1 Springs
Enter values:
Spring #2
$k1 = 800$

$x1o = 0$

$x1 = x$

Object 1: No Not a pulley
Yes moves in a straight line
0 force other than gravity, friction, springs, and normal

$m = 8$

$\theta g = -90$

$vo = 8.85438$

$v = 0$

$\Delta x = 2 + x$

$\Delta y = 0$

$n = n$

$\theta n = 90$

$\mu = 0.4$

$\theta f = 180$

Equations displayed:

$-313.6 = -400 \cdot x^2 - .4 \cdot n \cdot x - .8 \cdot n$

$0 = (n - 78.4) \cdot (x + 2)$

Continue

Solutions:

$x = .753729$

$n = 78.4$